

SUSY Breaking by Overlap of Wave Functions in Coexisting Walls

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Abstract

SUSY breaking without messenger fields is proposed. We assume that our world is on a wall and SUSY is broken only by the coexistence of another wall with some distance from our wall. The Nambu-Goldstone fermion is localized on the distant wall. Its overlap with the wave functions of physical fields on our wall gives the mass splitting of physical fields on our wall thanks to a low-energy theorem. We propose that this overlap provides a practical method to evaluate mass splitting in models with SUSY breaking due to the coexisting walls.

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Introduction

Supersymmetry (SUSY) is one of the most attractive scenarios to solve the hierarchy problem [1]. If SUSY is relevant to nature, it must be spontaneously broken at low energies since superparticles have not been observed yet. Therefore, it is very important to explore mechanisms of SUSY breaking and its mediation to our world.

Recently, much attention is paid to topological objects such as D-branes in string theories [2], BPS domain walls [3] and junctions [4] in supersymmetric field theories. In particular, “Brane World” scenario [5, 6], in which our four-dimensional space-time on these topological objects is embedded in higher dimensional space-time, has opened new directions to flavor physics, cosmology and astrophysics. It is well known that these topological objects break supersymmetries partially in general. In the light of this fact, it is interesting to study the mechanism of SUSY breaking and its mediation in the context of the brane world scenario.

The purpose of this paper is to propose a mechanism of SUSY breaking due to the coexistence of walls, and to show a practical way to evaluate the amount of SUSY breaking for our physical fields from the overlap with the wave function of the Nambu-Goldstone fermion. Our novel point is that the individual wall is supersymmetric (a BPS state [7]) and the SUSY breaking comes about only by the coexistence of these walls without any mediating bulk fields (messenger) to communicate SUSY breaking. We find that effective SUSY breaking scale observed on our wall becomes exponentially small as the distance between two walls grows, whereas the order parameter of the SUSY breaking is almost constant. We also find that mass splittings become larger for the higher massive modes.

If a wall is a BPS state, it usually breaks half of the SUSY of the original higher dimensional theory. Let us suppose that one of such 3-branes is our world and appropriate number of supercharges, say four supercharges remain intact. In addition to our brane, there may be other branes which preserves different combinations of supercharges. In such a situation, this $\mathcal{N} = 1$ SUSY on our brane is broken only because of the existence of the other distant branes. This will explain the smallness of the SUSY breaking on our brane. The simplest situation occurs when we have a wall (we refer to it as “our wall”) and an anti-wall (we refer to it as “the other wall”) parallel to each other. Although each wall preserves a half of supersymmetries in the theory, orthogonal combinations of supersymmetries are preserved by our wall and the other wall. Therefore the configuration as a whole breaks all of the supersymmetries. As we let the other wall going far away, its effect will not be felt by observers on our wall and the $\mathcal{N} = 1$ SUSY is restored. Therefore the smallness of the SUSY breaking can be attributed to the distance of the other wall. What is crucial for SUSY breaking is the *coexistence* of both walls. In other words, the supersymmetries on our wall is broken by the existence of the other wall distant from our wall.

Note that the SUSY breaking mechanism discussed here involves no complicated SUSY breaking sector on any of the walls. We emphasize that we need no bulk field (messenger) to communicate the SUSY breaking in contrast to Refs.[8]. An idea similar to ours has also been proposed and discussed briefly in Ref.[9].

Mass splitting and overlap integral

Here we propose a powerful method to find the SUSY breaking mass splitting caused by the coexistence of the other wall. If the SUSY is broken spontaneously, the Nambu-Goldstone fermion ψ_{NG} is contained in the supercurrent $J^\mu = \sqrt{2}i f \gamma^\mu \psi_{\text{NG}} + \dots$ with the strength given by the order parameter f of the SUSY breaking, which is the square root of the energy of the non-trivial background. Among the SUSY breaking terms, the low-energy effective Lagrangian contains a Yukawa coupling involving the Nambu-Goldstone fermion and a scalar a_m and spinor ψ_m fields that belong to the same supermultiplet

$$\mathcal{L}_{\text{Yukawa}} = h_{\text{eff}} a_m \psi_m \psi_{\text{NG}}. \quad (1)$$

The nonlinearly realized SUSY manifests itself as a low-energy theorem relating the Yukawa coupling h_{eff} to the squared mass splitting $\Delta m^2 \equiv m_B^2 - m_F^2$, where m_B and m_F are masses of a_m and ψ_m respectively [10]

$$h_{\text{eff}} = -\frac{\Delta m^2}{\sqrt{2}f}. \quad (2)$$

The Yukawa coupling h_{eff} is given by an overlap of wave functions of a_m and ψ_m localized on our wall and that of the Nambu-Goldstone fermion ψ_{NG} localized on the other wall. However, it is generally difficult to find the exact wave functions in the background of two or more walls. We will show that we can reliably evaluate an overlap integral of these wave functions in the extra dimension using an approximation for the wave functions in the single-wall background, even if it is difficult to find exact modes in the background of two walls. Improvement of the approximation is also proposed.

A model with wall and anti-wall

In the following, we consider simple toy models, three-dimensional walls in four-dimensional theory to illustrate our mechanism of SUSY breaking without inessential complications. Namely we will consider the direction $y = x^2$ as the extra dimension and compactify it on S^1 of radius R . To test the validity of our approximation methods by comparing with the exact result later, we will begin with a simple four-dimensional Wess-Zumino model with two real positive parameters g and Λ in which some wave

functions are known exactly⁵

$$\mathcal{L} = \Phi^\dagger \Phi|_{\theta^2 \bar{\theta}^2} + W(\Phi)|_{\theta^2} + \text{h.c.}, \quad W(\Phi) = \Lambda^2 \Phi - \frac{g}{3} \Phi^3, \quad (3)$$

where Φ is a chiral superfield whose components are defined as

$$\Phi(x^\mu + i\theta\sigma^\mu\bar{\theta}, \theta) = A(x^\mu + i\theta\sigma^\mu\bar{\theta}) + \sqrt{2}\theta\psi(x^\mu + i\theta\sigma^\mu\bar{\theta}) + \theta^2 F(x^\mu + i\theta\sigma^\mu\bar{\theta}).$$

This model has the following domain wall configuration as a classical solution[11]

$$A_{\text{cl}}(y) = \frac{k\omega}{g} \text{sn}(\omega(y - y_0), k), \quad \omega \equiv \frac{\sqrt{2g}\Lambda}{\sqrt{1+k^2}}, \quad (4)$$

where $\text{sn}(u, k)$ is the Jacobi's elliptic function and $0 \leq k \leq 1$. The period of this configuration is $4K(k)/\omega$, where $K(k)$ is the complete elliptic integral. Because of the $2\pi R$ periodicity in the extra dimension y , we take the case of $R = 2K(k)/\pi\omega$. Then the solution for $y_0 = 0$ represents two walls located at $y = 0$ and at $y = \pi R$. In the limit of $R \rightarrow \infty$, the wall at $y = 0$ becomes a BPS domain wall that preserves a half of the SUSY, and the other wall at $y = \pi R$ becomes another BPS domain wall preserving the other half of the SUSY. The latter may be regarded as an “anti-(domain) wall”. Here we will refer to the wall at $y = 0$ as “our wall” which means the wall we live on, and will call the wall at $y = \pi R$ as “the other wall” which gives the source of the small SUSY breaking effect on our wall.

Next we will consider the fluctuation modes around the background configuration (4). The bosonic mode functions $\phi_{an}(y)$ and $\phi_{bn}(y)$ with eigenvalues m_{an}^2 and m_{bn}^2 are defined in terms of the differential operators \mathcal{O}_{Ba} and \mathcal{O}_{Bb} as

$$\begin{aligned} \mathcal{O}_{Ba} &\equiv -\partial_y^2 - 2g(\Lambda^2 - 3gA_{\text{cl}}^2), \quad \mathcal{O}_{Bb} \equiv -\partial_y^2 + 2g(\Lambda^2 + gA_{\text{cl}}^2), \\ \mathcal{O}_{Ba}\phi_{an}(y) &= m_{an}^2\phi_{an}(y), \quad \mathcal{O}_{Bb}\phi_{bn}(y) = m_{bn}^2\phi_{bn}(y). \end{aligned} \quad (5)$$

Assuming the completeness of eigenfunctions $\phi_{an}(y)$ and $\phi_{bn}(y)$, the small real fluctuation fields a and b of the complex scalar field $A(x) = A_{\text{cl}}(y) + (a(x) + ib(x))/\sqrt{2}$ can be expanded as

$$a(x^m, y) = \sum_n \phi_{an}(y)a_n(x^m), \quad b(x^m, y) = \sum_n \phi_{bn}(y)b_n(x^m), \quad (m = 0, 1, 3). \quad (6)$$

Here $a_n(x^m)$ and $b_n(x^m)$ become scalar fields in three-dimensional effective theory with squared masses m_{an}^2 and m_{bn}^2 that are eigenvalues of \mathcal{O}_{Ba} and \mathcal{O}_{Bb} respectively.

Several light modes of $\phi_{an}(y)$ and $\phi_{bn}(y)$ are exactly known

$$\phi_{a,-1}(y) = C_{a,-1} \left\{ -\text{sn}^2(\omega y, k) + \frac{1 + k^2 + \sqrt{1 - k^2 + k^4}}{3k^2} \right\},$$

⁵We follow the convention in Ref.[12]

$$m_{a,-1}^2 = (1 + k^2 - 2\sqrt{1 - k^2 + k^4})\omega^2, \quad (7)$$

$$\phi_{a,0}(y) = C_{a,0}\text{cn}(\omega y, k)\text{dn}(\omega y, k), \quad m_{a,0}^2 = 0, \quad (8)$$

$$\phi_{a,1}(y) = C_{a,1}\text{sn}(\omega y, k)\text{dn}(\omega y, k), \quad m_{a,1}^2 = 3k^2\omega^2, \quad (9)$$

where $C_{a,n}$ are positive normalization factors and cn, dn are the elliptic functions. Other modes are heavier than these.

Since $m_{a,-1}^2 < 0$, the field $a_{-1}(x^m)$ is a tachyon. This instability corresponds to the wall-antiwall annihilation into the vacuum. However in the case of the large R , the tachyonic squared mass $m_{a,-1}^2$ is very close to zero, and the system is meta-stable. The massless field $a_0(x^m)$ is the Nambu-Goldstone (NG) field corresponding to the breaking of the translational invariance.

To expand small fluctuations of fermions into modes, we decompose the Weyl spinor ψ_α in four dimensions to two real two-component spinors $\psi^{(1)}$ and $\psi^{(2)}$, and define a coupled mode equation with an eigenvalue m_n

$$\psi_\alpha(x) = \frac{1}{\sqrt{2}}(\psi_\alpha^{(1)}(x) + i\psi_\alpha^{(2)}(x)), \quad \mathcal{O}_{F1}(y) \equiv (\partial_y + 2gA_{\text{cl}}(y)), \quad \mathcal{O}_{F2}(y) \equiv (-\partial_y + 2gA_{\text{cl}}(y)), \quad (10)$$

$$\mathcal{O}_{F1}\varphi_n^{(1)} = -m_n\varphi_n^{(2)}, \quad \mathcal{O}_{F2}\varphi_n^{(2)} = -m_n\varphi_n^{(1)}. \quad (11)$$

The four-dimensional fields $\psi^{(1)}$ and $\psi^{(2)}$ can be expanded by these eigenfunctions into three-dimensional fermion fields $\psi_n^{(1)}(x^m)$ and $\psi_n^{(2)}(x^m)$ with mass m_n

$$\psi^{(1)}(x^m, y) = \sum_n \varphi_n^{(1)}(y)\psi_n^{(1)}(x^m), \quad \psi^{(2)}(x^m, y) = \sum_n \varphi_n^{(2)}(y)\psi_n^{(2)}(x^m). \quad (12)$$

One can work out explicitly the zero modes $m_0 = 0$ with the normalization factor C_0

$$\varphi_0^{(1)}(y) = C_0\{k\text{cn}(\omega y, k) + \text{dn}(\omega y, k)\}^2, \quad \varphi_0^{(2)}(y) = C_0\{k\text{cn}(\omega y, k) - \text{dn}(\omega y, k)\}^2. \quad (13)$$

We can see that the mode functions $\varphi_n^{(1)}(y)$ and $\varphi_n^{(2)}(y)$ are localized at $y = 0$ and $y = \pi R$ respectively.

Let us decompose the four-dimensional supercharge $Q_\alpha \equiv (Q_\alpha^{(1)} + iQ_\alpha^{(2)})/\sqrt{2}$ into two two-component Majorana supercharges $Q^{(1)}$ and $Q^{(2)}$ which can be regarded as supercharges in three dimensions. On our wall $Q^{(2)}$ is broken, but $Q^{(1)}$ is conserved in the limit $R \rightarrow \infty$. Since $\psi_0^{(2)}(x^m)$ is the Nambu-Goldstone fermion corresponding to the broken supercharge $Q^{(1)}$, the massless field $\psi_0^{(2)}(x^m)$ is our Nambu-Goldstone fermion $\psi_{\text{NG}}(x^m)$ in Eq.(1). The mode function $\varphi_0^{(2)}(y)$ of the Nambu-Goldstone fermion is approximately localized on the other wall because $Q^{(1)}$ is broken primarily by the presence of the other wall. Similarly, the mode function $\varphi_0^{(1)}(y)$ is localized on our wall and $\psi^{(1)}(x^m)$ is the Nambu-Goldstone fermion corresponding to $Q^{(2)}$ breaking, whose property as a Nambu-Goldstone fermion is not of our primary concern here.

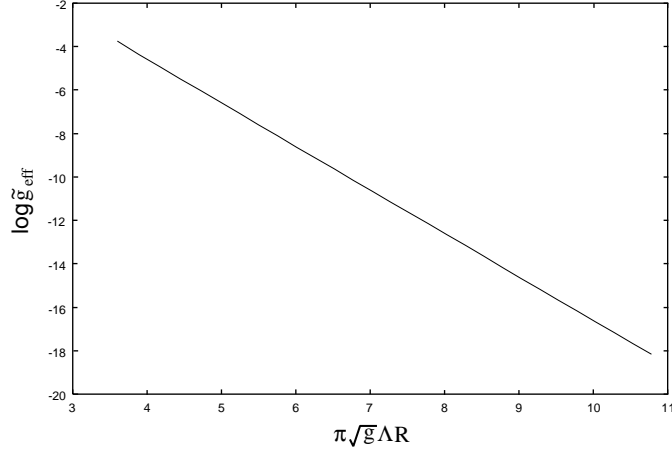


Figure 1: The normalized effective coupling \tilde{g}_{eff} as a function of the distance between the walls.

Rewriting the Lagrangian (3) in terms of the infinite towers of the three-dimensional fields in Eqs.(6) and (12) and integrating out the modes with positive mass squared, we obtain a three-dimensional low-energy effective Lagrangian for massless and tachyonic fields

$$\begin{aligned} \mathcal{L}^{(3)} = & -V_0 - \frac{1}{2}\partial^m a_{-1}\partial_m a_{-1} - \frac{1}{2}\partial^m a_0\partial_m a_0 - \frac{i}{2}\psi_0^{(1)}\not{\partial}\psi_0^{(1)} - \frac{i}{2}\psi_0^{(2)}\not{\partial}\psi_0^{(2)} \\ & - \frac{1}{2}m_{a,-1}^2 a_{-1}^2 + \sqrt{2}g_{\text{eff}}a_{-1}\psi_0^{(1)}\psi_0^{(2)}, \end{aligned} \quad (14)$$

where $\not{\partial} \equiv \gamma_{(3)}^m \partial_m$, $(\gamma_{(3)}^m) \equiv (-\sigma^2, i\sigma^3, -i\sigma^1)$ are γ -matrices in three dimensions and

$$V_0 \equiv \int_{-\pi R}^{\pi R} dy \{ \Lambda^4 - g^2 A_{\text{cl}}^4(y) \}, \quad (15)$$

$$g_{\text{eff}} \equiv g \int_{-\pi R}^{\pi R} dy \phi_{a,-1} \varphi_0^{(1)} \varphi_0^{(2)} = g C_0^2 (1 - k^2)^2 \int_{-\pi R}^{\pi R} dy \phi_{a,-1}. \quad (16)$$

The effective Yukawa coupling g_{eff} is proportional to an overlap integral of the mode functions localized on different walls. In Fig.1, we show the normalized effective coupling constant $\tilde{g}_{\text{eff}} \equiv g_{\text{eff}}/(g(\sqrt{g}\Lambda)^{\frac{1}{2}})$ which is a function of $\sqrt{g}\Lambda R$ only. We see that g_{eff} decays exponentially as the wall distance R increases.

Since our wall is approximately a BPS domain wall at large R , Eq.(14) becomes a three-dimensional supersymmetric lagrangian in the limit of $R \rightarrow \infty$ with $Q^{(1)}$ as the conserved supercharge. From the effective low-energy Lagrangian at finite R , we can find several terms which show the breaking of $Q^{(1)}$. Firstly we notice the Yukawa coupling g_{eff} without the associated scalar self-interactions. Secondly we see that the tachyonic squared mass $m_{a,-1}^2$ can also be used as the measure of the SUSY breaking since it represents the mass splitting of fermions and bosons. We find that the $m_{a,-1}^2$ also decay exponentially as R increases, and that the ratio of these two quantities $g_{\text{eff}}/m_{a,-1}^2$ becomes constant at large R as shown in Fig.2 where we used the “normalized” tachyonic mass squared $\tilde{m}_{a,-1}^2 \equiv m_{a,-1}^2/(g\Lambda^2)$ to make it dimensionless.

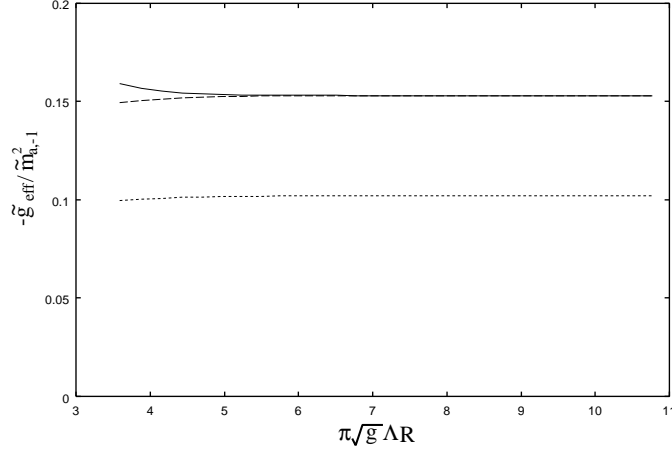


Figure 2: The ratio of the normalized effective coupling constant \tilde{g}_{eff} and the normalized tachyonic mass $\tilde{m}_{a,-1}^2$. The dotted line denotes the single-wall approximation, and the dashed line denotes the improved single-wall approximation.

This fact can be understood by noticing that g_{eff} is essentially one example of the coupling constant h_{eff} in Eq.(1). We define the following scalar mode functions

$$\phi_0^{(1)}(y) \equiv \frac{1}{\sqrt{2}}(\phi_{-1}(y) + \phi_0(y)), \quad \phi_0^{(2)}(y) \equiv \frac{1}{\sqrt{2}}(\phi_{-1}(y) - \phi_0(y)), \quad (17)$$

localized on our wall and the other wall respectively. By denoting the corresponding fields as $a_0^{(1)}(x^m) \equiv (a_{-1}(x^m) + a_0(x^m))/\sqrt{2}$ and $a_0^{(2)}(x^m) \equiv (a_{-1}(x^m) - a_0(x^m))/\sqrt{2}$, we can rewrite the last term in Eq.(14) involving the effective Yukawa interaction as

$$\mathcal{L}_{\text{yukawa}}^{(3)} = g_{\text{eff}}(a_0^{(1)}\psi_0^{(1)}\psi_0^{(2)} + a_0^{(2)}\psi_0^{(2)}\psi_0^{(1)}). \quad (18)$$

We can identify $(a_0^{(1)}, \psi_0^{(1)})$ as the supermultiplet with respect to approximate $Q^{(1)}$ -SUSY and $\psi_0^{(2)}$ as the Nambu-Goldstone fermion for the broken $Q^{(1)}$. Therefore the first term of Eq.(18) precisely gives the desired term⁶ in Eq.(1) with $a_0^{(1)}, \psi_0^{(1)}$ and $\psi_0^{(2)}$ identified as a_m, ψ_m and ψ_{NG} . We also rewrite the mass term in Eq.(14) in terms of $a_0^{(1)}$ and $a_0^{(2)}$ and find that the squared mass parameters for them are $m_{a,-1}^2/2$. Thus the low energy theorem in Eq.(2) now becomes using the inverse of the order parameter f of the SUSY breaking as

$$\frac{g_{\text{eff}}}{m_{a,-1}^2} = -\frac{1}{2\sqrt{2}f}. \quad (19)$$

The SUSY algebra implies that the order parameter of the $Q^{(1)}$ -SUSY breaking f is given by the positive square root of the vacuum (classical background) energy density V_0 in Eq.(15)

$$f^2 = \Lambda^4 \int_{-\pi R}^{\pi R} dy \left\{ 1 - \left(\frac{2k^2}{1+k^2} \right)^2 \text{sn}^4(\omega y, k) \right\}. \quad (20)$$

⁶ The remaining second term exhibits the case where our wall and the other wall is just interchanged.

We find that this order parameter satisfies the low-energy theorem (19) as anticipated and becomes $f \rightarrow \frac{4}{\sqrt{3}} \frac{\Lambda^2}{\sqrt{\sqrt{g}\Lambda}}$ in the limit of $R \rightarrow \infty$.

Practical method to obtain the mass splitting

Next we will consider the situation where some matter fields are localized on our wall. Since $Q^{(1)}$ -SUSY is preserved on our wall as $R \rightarrow \infty$, matter fields belong to supermultiplets of $Q^{(1)}$ -SUSY. Due to the existence of the other wall, however, $Q^{(1)}$ -SUSY is broken and mass splitting occurs between a boson and a fermion belonging to the same supermultiplet. As mentioned above, this mass splitting is related to the Yukawa coupling h_{eff} and the order parameter f of the SUSY breaking by the low-energy theorem in Eq.(2). Therefore we can find the mass splitting by calculating the effective Yukawa coupling h_{eff} . This method is powerful, since h_{eff} can be calculated approximately for large R by using the mode functions of a single wall and thus we can evaluate the mass splitting without solving the difficult problem of mass spectra in the background of two coexisting walls.

Let us first test our proposal to obtain the mass splitting by an approximate evaluation of the effective Yukawa coupling g_{eff} as an example. Since we are interested in the overlap of mode functions of supermultiplet of a boson and a fermion localized at our wall and that of the Nambu-Goldstone fermion, we need a good approximation for the mode functions in the vicinity of our wall at $y = 0$. Therefore we can safely take the supermultiplet mode functions in the supersymmetric limit of $R \rightarrow \infty$, namely in the single wall limit, instead of the exact mode functions. The bosonic mode functions defined in Eq.(17) and the fermionic ones in Eq.(13) are localized on our wall and becomes in this limit

$$\phi_0^{(1)}(y) = \varphi_0^{(1)}(y) = \frac{C}{\cosh^2(\sqrt{g}\Lambda y)}, \quad -\pi R \leq y \leq \pi R, \quad (21)$$

where C is a normalization factor for the interval $[-\pi R, \pi R]$.

As the first approximation for the mode function of the Nambu-Goldstone fermion, we are tempted to take the mode function in the presence of only one wall at $y = \pi R$ or $y = -\pi R$. However, to respect the symmetry under $y \rightarrow -y$ and the periodic structure, we take a superposition of half ($y \leq \pi R$) of the mode function for a single wall at $y = \pi R$ and another half ($y \geq -\pi R$) of the mode function for another single wall at $y = -\pi R$. We call this a “single-wall approximation”

$$\varphi_0^{(2)}(y) \approx \varphi_{0 \text{ single}}^{(2)}(y) = C \left(\frac{1}{\cosh^2(\sqrt{g}\Lambda(y - \pi R))} + \frac{1}{\cosh^2(\sqrt{g}\Lambda(y + \pi R))} \right), \quad -\pi R \leq y \leq \pi R. \quad (22)$$

Thus we obtain g_{eff} in Eq.(16) in this approximation as

$$g_{\text{eff}} \approx 2\sqrt{2}gC^3 \int_{-\pi R}^{\pi R} dy \frac{1}{\cosh^4(\sqrt{g}\Lambda y) \cosh^2(\sqrt{g}\Lambda(y - \pi R))}. \quad (23)$$

Since two of three modes involved in the Yukawa coupling have their peaks at $y = 0$, the behavior of the Nambu-Goldstone fermion mode function $\varphi_0^{(2)}$ around $y = 0$ is important to evaluate the above integral. Note that a fermionic zero mode $\varphi_0^{(2)}$ localized at $y = \pi R$ can be written [13] as

$$\varphi_0^{(2)}(y) = e^{2g \int^y dy' A_{\text{cl}}(y')}. \quad (24)$$

Hence to improve the approximation, we can use the superposition of two single walls as a background configuration A_{cl} in Eq.(24)

$$A_{\text{cl}}(y) \simeq \frac{\Lambda}{\sqrt{g}} \{-\tanh(\sqrt{g}\Lambda(y + \pi R)) + \tanh(\sqrt{g}\Lambda y) - \tanh(\sqrt{g}\Lambda(y - \pi R))\}, \quad -\pi R \leq y \leq \pi R. \quad (25)$$

We call this as the “improved” single-wall approximation and find

$$\varphi_0^{(2)}(y) \approx \varphi_0^{(2)}{}_{\text{improved}}(y) = C_0^{(2)} \frac{\cosh^2(\sqrt{g}\Lambda y)}{\cosh^2(\sqrt{g}\Lambda(y + \pi R)) \cosh^2(\sqrt{g}\Lambda(y - \pi R))}, \quad (26)$$

where $C_0^{(2)}$ is a normalization factor for the interval $[-\pi R, \pi R]$. In this improved single-wall approximation we obtain

$$g_{\text{eff}} \approx \sqrt{2}gC^2C_0^{(2)} \int_{-\pi R}^{\pi R} dy \frac{1}{\cosh^2(\sqrt{g}\Lambda(y + \pi R)) \cosh^2(\sqrt{g}\Lambda y) \cosh^2(\sqrt{g}\Lambda(y - \pi R))}. \quad (27)$$

The results of these approximations are shown in Fig.2. We see that the simplest single wall approximation already gives a correct order of magnitude estimate for g_{eff} including the sign. Therefore it gives a correct information for the mass splitting including which is heavier, boson or fermion. Moreover, the improved single-wall approximation gives a very accurate estimate of the mass splitting, for instance within 1 % in the case of $R \geq 5/(\sqrt{g}R)$.

To illuminate another aspect of our proposal, we next consider a matter chiral superfield $\Phi_{\text{m}} = A_{\text{m}} + \sqrt{2}\theta\psi_{\text{m}} + \theta^2 F_{\text{m}}$ interacting with Φ in the original lagrangian (3) through an additional superpotential $W_{\text{int}} = -h\Phi\Phi_{\text{m}}^2$. One can easily see that the linearized equations for the matter fields are identical to those for the field Φ building the wall except that the coupling g is replaced by h . By taking the coupling h bigger than g , we obtain several light modes of Φ_{m} localized on our wall. Let us decompose the matter fermion ψ_{m} into two real two-component spinors $\psi_{\text{mR}\alpha}$ and $\psi_{\text{mI}\alpha}$ as $\psi_{\text{m}\alpha} = (\psi_{\text{mR}\alpha} + i\psi_{\text{mI}\alpha})/\sqrt{2}$. Comparing it to Eq.(10), we find that the eigenvalue equations for $\psi_{\text{mR}\alpha}$ and $\psi_{\text{mI}\alpha}$ are obtained by replacing the coupling g by h in Eq.(11) for $\psi_{\alpha}^{(1)}$ and $\psi_{\alpha}^{(2)}$. For simplicity we consider the limit of $R \rightarrow \infty$ (i.e. single-wall approximation). We find that the low-lying eigenvalues are discrete at $m_{\text{mn}}^2 = g\Lambda^2 n(-n + 4h/g)$ with $n = 0, 1, 2, \dots < 2h/g$, and that the corresponding eigenfunctions $\rho_{\text{Rn}}(y)$ for the field $\psi_{\text{mR}\alpha}$ are

$$\rho_{\text{Rn}}(y) = \frac{N_n}{[\cosh(\sqrt{g}\Lambda y)]^{\frac{2h}{g}-n}} \text{F}\left(-n, 1-n+\frac{4h}{g}, 1-n+\frac{2h}{g}; \frac{1-\tanh(\sqrt{g}\Lambda y)}{2}\right), \quad (28)$$

m^2	0	13	24	33	40	45	48
Δm^2	1.4×10^{-7}	4.2×10^{-7}	5.4×10^{-7}	7.7×10^{-7}	1.3×10^{-6}	3.2×10^{-6}	6.5×10^{-5}

Table 1: The mass splitting at each mass level in the case that $h/g = 3.5$ and $R = 10/(\sqrt{g}\Lambda)$. Here m^2 is the squared mass of the supermultiplet and Δm^2 is the mass splitting between a boson and a fermion belonging to the same supermultiplet. The unit of the mass is $\sqrt{g}\Lambda$.

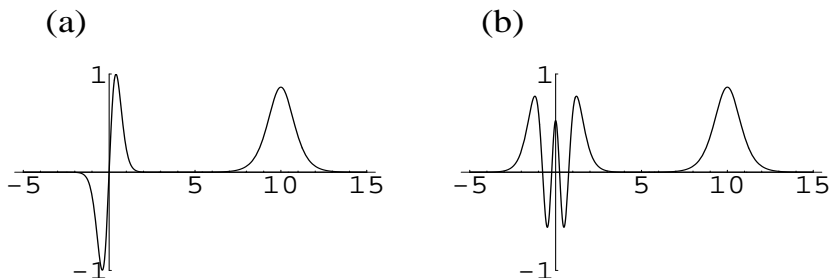


Figure 3: The profiles of massive modes and the Nambu-Goldstone fermion localized on the other wall in the case that $h/g = 3.5$ and $R = 10/(\sqrt{g}\Lambda)$. (a) is the mode of $m^2 = 13$ and (b) is the mode of $m^2 = 40$. The horizontal axis is the coordinate of y in unit of $1/(\sqrt{g}\Lambda)$.

where $F(\alpha, \beta, \gamma; z)$ is the hypergeometric function and N_n is a normalization factor. The mode functions for the field $\psi_{mI\alpha}$ can similarly be obtained from this ρ_R . Single-wall approximation allows us to use the SUSY to obtain the bosonic mode functions. By using these mode functions we can expand the fields

$$\psi_{mR}(x^m, y) = \sum_n \rho_{Rn}(y) \psi_{mRn}(x^m), \quad (29)$$

for instance, and similarly for other fields.

Let us apply our improved single-wall approximation in Eq.(26) for the Nambu-Goldstone field and to examine the mass splitting of massive matter fields. If R is large enough, the mode functions for the matter fields on our wall can be well-approximated by Eq.(28). In Table 1, we have shown the mass splitting at each mass level for a representative case of $h/g = 3.5$ and $R = 10/(\sqrt{g}\Lambda)$ and taking the unit of the mass as $1/(\sqrt{g}\Lambda)$. We notice that mass splitting becomes larger for heavier supermultiplets. As can be seen from Fig.3, heavier modes have wider profile in the extra dimension and have a larger overlap with the Nambu-Goldstone fermion, which is localized on the other wall. Since this is likely to be a generic feature of higher massive modes, we expect this phenomenon to be generic.

A model with two fields

The wall-antiwall system (4) in the previous model has a tachyonic mode signaling the annihilation of wall-antiwall into vacuum. In the following, we will give an example of two walls preserving different halves of SUSY which cannot be annihilated into the vacuum. The model is a Wess-Zumino model with

two chiral superfields Φ and X , and its superpotential is

$$W(\Phi, X) = \frac{m^2}{\lambda}\Phi - \frac{\lambda}{3}\Phi^3 - \alpha\Phi X^2, \quad (30)$$

where m is a mass parameter and λ and α are dimensionless coupling constants. This model has four degenerate SUSY vacua and is known to have several types of domain wall configurations [14, 15]. Unlike the previous model, we do not compactify the direction of y here.

BPS domain wall solutions can be obtained from the following first-order equations

$$\frac{d\phi}{dy} = \Omega \left(\frac{m^2}{\lambda} - \lambda\phi^2 - \alpha\chi^2 \right), \quad \frac{d\chi}{dy} = \Omega(-2\alpha\phi\chi), \quad (31)$$

where ϕ and χ are real parts of scalar components of Φ and X respectively, and Ω is a phase factor determined by boundary conditions. For the case of $\alpha = \lambda/4$, Eq.(31) has analytic solutions representing a wall localized at $y = 0$

$$\Omega = 1 : \quad \phi_{\text{cl}}^{(1)}(y) = \frac{m}{2\lambda} \left(1 + \tanh \frac{my}{2} \right), \quad \chi_{\text{cl}}^{(1)}(y) = \frac{\sqrt{2}m}{\lambda} \sqrt{1 - \tanh \frac{my}{2}}, \quad (32)$$

which preserves a half of the SUSY called $Q^{(1)}$. This will be referred as type I and

$$\Omega = -1 : \quad \phi_{\text{cl}}^{(2)}(y) = \frac{m}{2\lambda} \left(1 - \tanh \frac{my}{2} \right), \quad \chi_{\text{cl}}^{(2)}(y) = -\frac{\sqrt{2}m}{\lambda} \sqrt{1 + \tanh \frac{my}{2}}, \quad (33)$$

which preserves the other half of the SUSY and will be called type II.

Now we consider a non-BPS configuration constructed from a superposition of these BPS walls of type I at $y = 0$ (our wall) and type II at $y = a$ (the other wall) [15]

$$\begin{aligned} \phi_{\text{cl}}(y) &= \frac{m}{2\lambda} \left(\tanh \frac{my}{2} - \tanh \frac{m(y-a)}{2} \right), \\ \chi_{\text{cl}}(y) &= \frac{\sqrt{2}m}{\lambda} \left(\sqrt{1 - \tanh \frac{my}{2}} - \sqrt{1 + \tanh \frac{m(y-a)}{2}} \right). \end{aligned} \quad (34)$$

Note that this is not a static classical solution of the equations of motion. However, it is approximately a static classical solution for large values of a , which is a distance between two walls. In addition, two walls cannot annihilate into a vacuum, since the vacua at $y = -\infty$ and $y = \infty$ are different. Therefore we have no reason to suspect tachyonic modes.

We will focus on (nearly) massless modes localized on our wall at $y = 0$, and evaluate the SUSY breaking mass splitting between them by calculating the Yukawa coupling in the three-dimensional effective theory. Integrating over the y -direction and integrating out the massive modes, we obtain a three-dimensional effective Lagrangian. It contains the Yukawa coupling involving $a_0^{(1)}$ and $\psi_0^{(1)}$, which form a $Q^{(1)}$ -supermultiplet and localized on our wall, and the Nambu-Goldstone fermion ψ_{NG} associated with the $Q^{(1)}$ -SUSY breaking

$$\mathcal{L}_{\text{yukawa}}^{(3)} = g_{\text{eff}} a_0^{(1)} \psi_0^{(1)} \psi_{\text{NG}},$$

$$g_{\text{eff}} = -\frac{\lambda}{2\sqrt{2}} \int_{-\infty}^{\infty} dy (4\rho^2 \rho_{\text{NG}} + 2\rho\sigma\sigma_{\text{NG}} + \sigma^2 \rho_{\text{NG}}), \quad (35)$$

where ρ and σ are the ϕ and χ -components of the common mode function of the supermultiplet $(a_0^{(1)}, \psi_0^{(1)})$, and ρ_{NG} and σ_{NG} are the ϕ and χ -components of the Nambu-Goldstone fermion mode function. Using the single-wall approximation, we obtain mode functions for the supermultiplet ρ, σ and for the Nambu-Goldstone fermion $\rho_{\text{NG}}, \sigma_{\text{NG}}$

$$\rho(y) = \frac{C}{\cosh^2 \frac{my}{2}}, \quad \sigma(y) = -\sqrt{2}C \left(1 + \tanh \frac{my}{2}\right) \sqrt{1 - \tanh \frac{my}{2}}, \quad (36)$$

$$\begin{aligned} \rho_{\text{NG}}(y) &= \frac{C}{\cosh^2 \frac{m(y-a)}{2}}, \\ \sigma_{\text{NG}}(y) &= -\sqrt{2}C \left(1 + \tanh \frac{m(y-a)}{2}\right) \sqrt{1 - \tanh \frac{m(y-a)}{2}}, \end{aligned} \quad (37)$$

where C is a normalization factor and is taken to be positive.

Substituting these mode functions into Eq.(35), we obtain the value of the effective Yukawa coupling $g_{\text{eff}} \approx 0.520\lambda\sqrt{m}e^{-0.50am}$ for $am \gg 1$. From the energy density of the background field configuration we obtain also the SUSY breaking order parameter $f \approx 2\sqrt{2/3}(m^{3/2}/\lambda)$. Therefore we find that the mass splitting between boson and fermion as $\Delta m^2 = m_{\text{B}}^2 - m_{\text{F}}^2 \approx 1.20m^2e^{-0.50am}$. The fact that the mass squared of the bosonic mode shifts to positive values⁷ is in accord with our observation of the absence of tachyon corresponding to the annihilation into the vacuum.

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⁷The fermionic zero mode remains massless due to its property of the Nambu-Goldstone fermion.

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